5.2 Matrix Solutions of Linear Systems

In the previous lesson we reviewed substitution and elimination to solve linear systems. Both methods will always work, but elimination is generally faster, especially for higher numbers of variables. We can do elimination without writing the variables if we are careful about the original structure. This process is similar to the way we performed synthetic division without writing $x^{2}, x$, etc.

For example, consider the system:

$$x+3y+2z=1$$

$$2x+y-z=2$$

$$x+y+z=2$$

The augmented matrix for this system is: $\begin{matrix}1&3&2\\2&1&-1\\1&1&1\end{matrix} \begin{matrix}1\\2\\2\end{matrix}$ . We simply write the coefficients of each term and remember the first column represents $x$, 2nd column $y$, 3rd column $z$, an equal sign between the 3rd and 4th columns, 4th column constants, and plus signs between all 3 of the first columns.

Now we perform the elimination like before. This process is known as the Gauss-Jordan elimination method. We’ll come back to this one. First, let’s work through some simpler examples.

**Practice:** Given the system

 $3x-2y=5$

 $4x+y=3$

1. Write the augmented matrix.
2. How many rows does the matrix have?
3. How many columns does the matrix have?
4. What is the dimension of the matrix?
5. What number must the 2nd row be multiplied by if we are planning to eliminate the $y$ variable by addition?
6. What complete row operation will accomplish this?
7. Perform this row operation and write the resulting matrix.
8. Similarly, eliminate the $x$ variable.
9. What is the solution of the system?
10. Check the solution.

**Practice:** Answer the same questions for the system

 $-5x+2y=4$

 $-10x+4y=-6$

**Practice:** Answer the same questions for the system

 $2x-y=2$

 $8x-4y=8$

How can you tell whether the linear system will have one solution, no solution, or infinitely many solutions? How do you write the solution set in each case?

**Practice:** Solve the first system using the Gauss-Jordan method.

As you see, this process can be rather detailed, especially for large numbers of variables. Thankfully, this process can be programmed into a calculator or computer. Simply enter the augmented matrix into your calculator, then select the “rref” command, which stands for reduced row echelon form. Your system will be solved. Let’s practice a few.

Solve by matrix methods with a calculator. If the solution is infinite, write it with the last variable arbitrary.

Redo the first example with a calculator. Then solve each system below with a calculator.

$$x-y+5z=-6$$

$$3x+3y-z=10$$

$$x+3y+2z=5$$

$$x+y=2$$

$$2x+2y=4$$

$$x-3y+2z+5=0$$

$$3x+5y-8z+1=0$$

$$4x+2y-6z+8=0$$

$$x-3y+2z+5=0$$

$$3x+5y-8z+1=0$$

$$4x+2y-6z+6=0$$