4.4 Evaluating Logarithms and Change of Base

Two bases are very frequently used with Exponential and Logarithmic functions. Thus, shorthand notations are used for them.

**Common Logarithm:** We will define . Since base is what we base our number system on, those logarithms are very common. If we don’t write a base, it is understood to be .

**Natural Logarithm:** Similarly, we define . Base exponentials and logarithms have some very important properties that won’t be appreciated until you take Calculus, but they are also frequently used. The is French in origin and roughly translates to “Logarithm Natural”. French is similar to Spanish in that the adjectives are written after the nouns, not before like in English.

Both of the above functions are easy to find on your calculator.

**Change of Base Theorem:** if all 3 of these terms are defined.

This is a useful theorem in that it allows us to write all exponential and logarithmic functions in any base that we choose. Of course, we will often choose to use base or base .

Proof:

* Let
* , definition of logarithm
* , take the base logarithm of both sides
* , Logarithm property 3
* , Substitution
* , Division

Practice: Write using only common logarithms, then again using only natural logarithms. Then calculate each of these and round to 4 decimal places.

Do the same for

Example Problems

Calculate the following to the thousandths place. Check them with exponential functions.

Application Problems

In Chemistry, the of a solution is defined to be where is the hydronium ion concentration: The ratio of the number of moles of hydronium ions to the number of liters of solution. Water will naturally gain protons to become or lose protons to become (hydroxide). Based on the number of these changes, one can state how acidic a solution is. You’ll learn more about this when you take Chemistry. The lower the , the more acidic, the higher the , the more basic. The range of the function is to with a being a strong acid and a being a strong base.

* Baking soda , a base, has a hydronium ion concentration of . What is the of baking soda?
* Pure water has a hydronium ion concentration of . Coincidentally, it also has a hydroxide ion concentration of . What is the of pure water?
* Hydrochloric Acid and Sulfuric Acid are both strong acids. What is their hydronium ion concentration?
* A human stomach has a (approximate). What is the hydronium ion concentration in a human stomach?
* Can you explain why nearly all of the food and drink you consume is acidic? What would happen if you consumed foods that are basic?