4.2 Exponential Functions

**Definition:** Let $a>0$ be a positive real number that is not equal to $1$. We say that $f\left(x\right)=a^{x}$ is an exponential function with a base of $a$.

A function where the base is constant and the exponent is a variable is called an exponential function. A function in which the base is variable, but the exponent is constant is called a power function. For example, $f\left(x\right)=5^{x}$ is an exponential function whereas, $f\left(x\right)=x^{5}$ is a power function.

We will restrict our study of exponential functions to those with only real exponents.

Practice: Graph each of the following.

* $f\left(x\right)=2^{x}$
* $f\left(x\right)=3^{x}$
* $f\left(x\right)=(\frac{1}{2})^{x}$
* Use the properties of exponents that you reviewed in R.6 to rewrite this function with a negative exponent. Can you now describe the geometrical transformation that $f\left(x\right)=(\frac{1}{2})^{x}$ is from $f\left(x\right)=2^{x}$?

What is the domain and range of all exponential functions of the form $\left(x\right)=a^{x}$ ?

Do exponential functions have any asymptotes? If so, what kind and what is the equation?

Are exponential functions one to one?

Keep in mind that if these functions are translated up or down or reflected across the $x$-axis, then the range and asymptotes may change.

What is the range and horizontal asymptote of $\left(x\right)=-3^{x}+5$ ?

Sketch the graph of this function

Practice: Let $f\left(x\right)=5^{x}$. Evaluate the following.

* $f(0)$
* $f(1)$
* $f(-1)$
* $f(\frac{1}{2})$
* $f(\frac{5}{2})$

Solving Exponential Equations (Introduction)

 The strategy (for now) is to rewrite each expression with the same base, then equate the exponents. This process is not always easy, clear, or the best approach, but luckily we will learn a more general technique in the near future.

Practice: Solve each equation

* $2^{x+3}=4^{x-1}$
* $81^{2x-3}=27^{x+1}$
* $36^{x-3}=6$
* $(\frac{2}{3})^{x}=\frac{9}{4}$
* $(\frac{1}{9})^{x}=27$

New Number

* The real number $e≈2.72$ is an irrational number named after Leonard Euler. Its definition is not needed at this time.

Solve:

* $e^{-3x}=\frac{1}{\sqrt{e}}$
* $e^{x+2}=\frac{1}{e^{3x}}$
* $(\frac{1}{e})^{-3x}=(\frac{1}{e^{3}})^{x+2}$

Sometimes we can solve exponential equations by raising both sides to a power. We’ve done some like this previously, these are just review.

* $r^{^{5}/\_{4}}=32$
* $y^{^{2}/\_{3}}=9$