3.2 Notes: Synthetic Division

Our motivation today is division of polynomial functions. Let’s start with a review of elementary division of integers.

Recall the **Division Algorithm for Integers**: If and are integers, then there exist unique integers and with such that . We call the dividend, the divisor, the quotient, and the remainder.

For example, if and , we can write . See if you understand why this is also true: .

Practice: Write each of the following in the form where

* ,

The division algorithm defines the division process. Polynomial division is just a more general case of this process. Given polynomials and , there exist unique polynomials and with the degrees of and less than the degree of such that . We define the dividend , quotient , divisor , and remainder similarly.

Example:

Review the long division process by dividing by .

Practice: Divide by

Practice: Divide by

Notice that in the division algorithm if the divisor is a linear factor say , then the remainder will be a constant (0 degree polynomial). Thus, we can invent a process called synthetic division based on multiplication and addition rather than multiplication and subtraction by switching to and using boxes as place holders for each lower degree term. This process is hard to verbalize so let’s do a few examples. **Caution:** This process only works if the divisor is linear. A divisor of or something similar is better suited to long division.

Practice: Divide by using synthetic division

Practice: Perform the division

Practice: Let and . Write in the form where is a polynomial function and is a real number.

Divide by . How do we deal with the coefficient in the divisor? This is important to understand.

Let and . Determine and such that

Express in the form for the given value of . Let ;

So far we have considered only real divisors, but this process also works for complex divisors.

Practice: Divide by

**Remainder Theorem** – When a polynomial is divided by , the remainder is

Proof:

Practice: Use the remainder theorem to find for ,

**Definition** – The number is a zero of a polynomial function if and only if .

Practice: Is the given value of a zero of the polynomial function? If not, state the value of . ,

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Hopefully you see that our Arabic numeral system is a specific case of the field of polynomials with . We’re studying the most general set of polynomials in which the value of or base could be anything (including complex numbers). In computer science, the bases of , , and are commonly used.

Show that the first example of is equivalent to the more general problem when .