2.7 Graphing Techniques

**Purpose:** The functions that we learned to graph in the previous lesson can be transformed geometrically to produce just about every graph we would be interested in making. Recall the specific transformations of Translation, Reflection, Rotation, and Dilation that you learned in your Geometry class. We will see a relationship between these transformations and the types of equations that generate them.

Vertical Translations

Investigate the functions:

* $y\_{1}=|x|$
* $y\_{2}=|x|+3$
* $y\_{3}=|x|-2$

How are they different? How are they similar?

In general, if $f(x)$ has a graph, then $f\left(x\right)+k$ will produce a graph that is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Horizontal Translations

Similarly, consider:

* $y\_{1}=|x|$
* $y\_{2}=|x+3|$
* $y\_{3}=|x-2|$

In general, if $f(x)$ has a graph, then $f\left(x-h\right)$ will produce a graph that is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Reflections

Consider:

* $y\_{1}=\sqrt{x}$
* $y\_{2}=-\sqrt{x}$
* $y\_{3}=\sqrt{-x}$

In general, if $f(x)$ has a graph, then $f\left(-x\right)$ will produce a graph that is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

In general, if $f(x)$ has a graph, then $-f\left(x\right)$ will produce a graph that is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Dilations in the Vertical Plane

Consider:

* $y\_{1}=\sqrt{x}$
* $y\_{2}=2\sqrt{x}$
* $y\_{3}=\frac{1}{3}\sqrt{x}$

In general, if $f(x)$ has a graph and $a>1$, then $a∙f\left(x\right)$ will produce a graph that is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

In general, if $f(x)$ has a graph and $0<a<1$, then $a∙f\left(x\right)$ will produce a graph that is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Dilations in the Horizontal Plane

Consider:

* $y\_{1}=\sqrt{x}$
* $y\_{2}=\sqrt{2x}$
* $y\_{3}=\sqrt{\frac{x}{3}}$

In general, if $f(x)$ has a graph and $a>1$, then $f\left(ax\right)$ will produce a graph that is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

In general, if $f(x)$ has a graph and $0<a<1$, then $f\left(ax\right)$ will produce a graph that is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Geometry Review

A graph is symmetric with respect to the $x-$axis if and only if the $x-$axis is a line of symmetry. Symmetric with respect to the $y-$axis is defined similarly.

A graph is symmetric with respect to the origin if and only if it has $180°$ rotational symmetry. Recall that a $180°$ rotation is equivalent to composite reflections across the $x$ and $y$ axes.

Even and Odd Definitions

Let $f$ be a real function. Then,

* $f$ is an even function if and only if $f\left(-x\right)=f(x)$ for all $x$ in the domain of $f$.
* $f$ is an odd function if and only if $-f\left(-x\right)=f(x)$ for all $x$ in the domain of $f$. Note that the textbook uses an equivalent definition of $f\left(-x\right)=-f(x)$. I think this definition is maybe easier to write, but makes the geometrical symmetries appear less clear.
* The geometric interpretation of the above definitions should be clear now.
	+ Even functions are symmetric about the $y-$axis since a reflection across the $y-$axis will produce the same graph.
	+ Odd functions are symmetric about the origin ($180°$ rotational symmetry) since a composite reflection across both the $x$ and $y$ axes will produce the same graph.

Examples: Determine if the following functions are even, odd, or neither.

* $f\left(x\right)=2x+4$
* $f\left(x\right)=x^{3}+x$
* $f\left(x\right)=x^{2}+4$

**Note:** Some transformations are not unique. For example,

$$y=\sqrt{4x}=2\sqrt{x}$$

Thus, the square root graph can be stretched by a factor of $2$ vertically or shrunk by a factor of $4$ horizontally and both graphs will be equivalent.

Another example of this would be $y=2x-4=2(x-2)$. The identity function (a line) stretched vertically by a factor of $2$ and translated down $4$ is the same as if the line were first translated right $2$, then stretched vertically by a factor of $2$. Take a moment to graph both of these so you understand the equivalence.

How you describe and understand the transformations is your choice to make and may be different than your peers.

Examples

Describe the transformations of $f\left(x\right)=\sqrt[3]{x}$ that will produce the graph of the following and produce a sketch of their graphs.

* $g\left(x\right)=-3\sqrt[3]{x+2}$
* $h\left(x\right)=\sqrt[3]{-x+3}+5$
* $k\left(x\right)=\sqrt[3]{\frac{1}{8}x-\frac{1}{4}}$

Example: Suppose that $(3, 8)$ is on the graph of $y=f(x)$. Find a point that lies on each of the following graphs.

* $f(x+3)$
* $f\left(x\right)+3$
* $f(3x)$
* $3f(x)$
* $f(\frac{1}{2}x)$
* $f(-x)$
* $-f(x)$