4.3 Logarithmic Functions

Are exponential functions one to one?

Since they are one to one, their inverse exists. We define the inverse of an exponential to be a logarithm.

**Definition:** Let $f\left(x\right)=a^{x}$ for $a>0$ and $a\ne 1$. Then the inverse of $f$, denoted $f^{-1}\left(x\right)=log\_{a}(x)$ is the base $a$ logarithm function.

Make a table of values for $y=2^{x}$. Do the same for $y=log\_{2}x$. What is the domain and range of each function? What do the graphs look like? Are there asymptotes?

Sketch the graph of $y=log\_{\frac{1}{2}}(x)$

Sketch the graph of $y=-2log\_{3}\left(x+4\right)-3$

Practice: Evaluate each logarithm. Do not use a calculator.

* $log\_{3}81$
* $log\_{3}\sqrt{3}$
* $log\_{3}\frac{1}{9}$
* $log\_{3}27^{^{2}/\_{5}}$
* $log\_{3}1$

Practice: Rewrite each equation in exponential form. Then write the solution set.

* $log\_{x}\left(81\right)=4$
* $log\_{2}\sqrt{2}=x$
* $log\_{3}\left(x\right)=-2$
* $log\_{49}\sqrt[3]{7}=x$

Properties of Logarithms. Let $a>0$, $a\ne 1$, $x>0$, $y>0$, and $r\in R$. Then,

* $log\_{a}\left(x\right)+log\_{a}\left(y\right)=log\_{a}(xy)$
* $log\_{a}\left(x\right)-log\_{a}\left(y\right)=log\_{a}(\frac{x}{y})$
* $log\_{a}\left(x^{r}\right)=r∙log\_{a}(x)$

Notice the similarity (with respect to inverses) that these properties have with the exponential properties you should be familiar with.

* $a^{x}a^{y}=a^{x+y}$
* $\frac{a^{x}}{a^{y}}=a^{x-y}$
* $(a^{x})^{y}=a^{xy}$

Practice. Rewrite each expression as the logarithm of a single quantity.

* $\frac{2}{3}log\_{x}\left(mp^{2}\right)+3log\_{x}(m^{3}p)$
* $log\_{7}\left(8x^{2}\right)-log\_{7}(2x)$

Practice. Expand each logarithm.

* $log\_{10}⁡(\frac{3x^{2}}{5})$

Summary of Logarithm and Exponential Inverse Relationship.

It should now be clear that

$$a^{log\_{a}(x)}=log\_{a}a^{x}=x$$

This is the inverse definition in a nutshell. Exponentials are the inverse of logarithms and logarithms are the inverse of exponentials.

Evaluate

* $7^{log\_{7}(13)}=$
* $8^{log\_{2}(5)}=$
* $log\_{3}3^{2.15}=$
* $log\_{\frac{1}{3}}3^{√π}=$

All of the above should be done without using a calculator.

Around the mid 1980’s, scientific calculators became common and affordable. At that time, all one needed to do to determine logarithms was to type them into a calculator. Before that time, one had to look up the values that someone had calculated in a table, which probably only showed base $10$ logarithms and probably only integer values between $1$ and $10$. You then had to calculate the logarithm you needed from these values using the properties we listed above. For instance, a table might show that $log\_{10}\left(3\right)=.477$ and that $log\_{10}\left(2\right)=.301$.

Using only these values, calculate

* $log\_{10}(12)$
* $log\_{10}(\frac{9}{2})$
* $log\_{10}(2400)$