4.3 Logarithmic Functions

Are exponential functions one to one?

Since they are one to one, their inverse exists. We define the inverse of an exponential to be a logarithm.

**Definition:** Let for and . Then the inverse of , denoted is the base logarithm function.

Make a table of values for . Do the same for . What is the domain and range of each function? What do the graphs look like? Are there asymptotes?

Sketch the graph of

Sketch the graph of

Practice: Evaluate each logarithm. Do not use a calculator.

Practice: Rewrite each equation in exponential form. Then write the solution set.

Properties of Logarithms. Let , , , , and . Then,

Notice the similarity (with respect to inverses) that these properties have with the exponential properties you should be familiar with.

Practice. Rewrite each expression as the logarithm of a single quantity.

Practice. Expand each logarithm.

Summary of Logarithm and Exponential Inverse Relationship.

It should now be clear that

This is the inverse definition in a nutshell. Exponentials are the inverse of logarithms and logarithms are the inverse of exponentials.

Evaluate

All of the above should be done without using a calculator.

Around the mid 1980’s, scientific calculators became common and affordable. At that time, all one needed to do to determine logarithms was to type them into a calculator. Before that time, one had to look up the values that someone had calculated in a table, which probably only showed base logarithms and probably only integer values between and . You then had to calculate the logarithm you needed from these values using the properties we listed above. For instance, a table might show that and that .

Using only these values, calculate