1.6 Other Types of Equations: Rational, Radical, and Quadratic Substitutions

Key things to remember for this lesson:

* is undefined. We cannot divide by , ever.
* If an equation has fractions, it is best to start by eliminating the fractions. Do this by multiplying each side of the equation by the of the denominators. For example, if we were to solve we would notice that . In other words, is the smallest number that is a multiple of . Now multiply both sides of the equation by to get . After simplification, this becomes . Then solve to get .
* If we take an even root while solving an equation, we must remember that there are two possible solutions, a positive and a negative. For example, if , then the solutions are . However, if we take an odd root, then there is only one solution. If , then . is not a solution to the 2nd equation. Remember that roots are often written as fractional exponents. involves taking the th root. So if is even, it is an even root. If is odd, it is an odd root.

Solve each equation. For rational equations such as these, the technique involves 3 steps:

* Eliminate the fractions. You may need to factor the denominators to see the .
* Solve. The resulting equations are usually linear or quadratic.
* Check all possible solutions to make sure that you are not dividing by .

For radical equations such as these, the technique to follow is:

* Isolate the radical term (get it alone on one side of the equation). If there are multiple radical terms, isolate one of them and don’t worry about the others yet.
* Raise both sides to a power to eliminate the radical. If the power is fractional, you must remember that even roots require you insert the symbol while odd roots do not.
* Solve the resulting equation. If radical terms remain after doing this, repeat the first 2 steps until all radicals are eliminated.
* Check solutions. It is very difficult to tell ahead of time whether a proposed solution is correct or not, so the safest way to deal with this is to always check the solutions in the original equation.

Equations that are quadratic in form. Follow this technique:

* Identify the variable that is quadratic. Make a substitution for this variable. For example, if , we should recognize that . Then we let and rewrite the original equation as .
* Solve the resulting quadratic equation for .
* Go back to the substitution and solve for the original variable.

Equations that are symbolic. Use the same techniques discussed earlier.

1. . Solve for
2. . Solve for
3. . Solve for