1.6 Other Types of Equations: Rational, Radical, and Quadratic Substitutions

Key things to remember for this lesson:

* $\frac{a}{0}$ is undefined. We cannot divide by $0$, ever.
* If an equation has fractions, it is best to start by eliminating the fractions. Do this by multiplying each side of the equation by the $LCM$ of the denominators. For example, if we were to solve $\frac{2}{3}-\frac{1}{5}x=\frac{5}{6}$ we would notice that $LCM\left(3,5,6\right)=30$. In other words, $30$ is the smallest number that is a multiple of $3, 5, 6$ . Now multiply both sides of the equation by $30$ to get $30(\frac{2}{3}-\frac{1}{5}x)=30(\frac{5}{6})$. After simplification, this becomes $20-6x=25$. Then solve to get $x=-\frac{5}{6}$.
* If we take an even root while solving an equation, we must remember that there are two possible solutions, a positive and a negative. For example, if $x^{2}=36$, then the solutions are $x=\pm \sqrt{36}=\pm 6$. However, if we take an odd root, then there is only one solution. If $x^{3}=27$, then $x=\sqrt[3]{27}=3$. $x=-3$ is not a solution to the 2nd equation. Remember that roots are often written as fractional exponents. $x^{^{m}/\_{n}}$ involves taking the $n$th root. So if $n$ is even, it is an even root. If $n$ is odd, it is an odd root.

Solve each equation. For rational equations such as these, the technique involves 3 steps:

* Eliminate the fractions. You may need to factor the denominators to see the $LCM$.
* Solve. The resulting equations are usually linear or quadratic.
* Check all possible solutions to make sure that you are not dividing by $0$.
1. $\frac{x}{x-5}+\frac{5}{x-5}=5$
2. $\frac{2}{x+1}+\frac{1}{x-2}=-\frac{6}{x^{2}-x-2}$
3. $\frac{2}{x^{2}+7x+10}-\frac{1}{x^{2}-x-6}=\frac{3}{x^{2}+4x-5}$
4. $\frac{x}{x-4}-4=\frac{4}{x-4}$
5. $\frac{-2}{x-3}+\frac{3}{x+3}=\frac{-12}{x^{2}-9}$
6. $\frac{2x-5}{x}=\frac{x-2}{3}$
7. $\frac{4x+3}{x+1}+\frac{2}{x}=\frac{1}{x^{2}+x}$

For radical equations such as these, the technique to follow is:

* Isolate the radical term (get it alone on one side of the equation). If there are multiple radical terms, isolate one of them and don’t worry about the others yet.
* Raise both sides to a power to eliminate the radical. If the power is fractional, you must remember that even roots require you insert the $\pm $ symbol while odd roots do not.
* Solve the resulting equation. If radical terms remain after doing this, repeat the first 2 steps until all radicals are eliminated.
* Check solutions. It is very difficult to tell ahead of time whether a proposed solution is correct or not, so the safest way to deal with this is to always check the solutions in the original equation.
1. $\sqrt{2x+1}=-3$
2. $\sqrt[3]{x+1}-5=-3$
3. $\sqrt{3\sqrt{2x+3}}=\sqrt{5x-6}$
4. $\sqrt[4]{x-15}=2$
5. $\sqrt{x}+2=\sqrt{4+7\sqrt{x}}$
6. $\sqrt{x+5}-\sqrt{2x}=\sqrt{x-3}$
7. $x^{{5}/{4}}=32$
8. $x^{2/3}=9$
9. $(x-3)^{\frac{2}{5}}=4$
10. $3x^{\frac{3}{4}}=x^{\frac{1}{2}}$

Equations that are quadratic in form. Follow this technique:

* Identify the variable that is quadratic. Make a $u$ substitution for this variable. For example, if $x^{6}-3x^{3}+2=0$, we should recognize that $x^{6}=(x^{3})^{2}$. Then we let $u=x^{3}$ and rewrite the original equation as $u^{2}-3u+2=0$.
* Solve the resulting quadratic equation for $u$.
* Go back to the $u$ substitution and solve for the original variable.
1. $2x^{4}-7x^{2}=-5$
2. $(2x+5)^{{2}/{5}}-3(2x+5)^{{1}/{5}}+2=0$
3. $x^{-2}+3x^{-1}=10$

Equations that are symbolic. Use the same techniques discussed earlier.

1. $d=k\sqrt{h}$. Solve for $h$
2. $\frac{1}{R}=\frac{1}{r\_{1}}+\frac{1}{r\_{2}}$. Solve for $r\_{1}$
3. $a^{2}+b^{2}=c^{2}$. Solve for $a$