1.5 Notes: Applications of Quadratic Equations

Last time we looked at how to solve quadratic equations. This time, we’ll discuss some applications where quadratic equations appear.

Geometry Problems. Below are some important facts that you need to remember.

* Pythagorean Theorem. If a triangle is right with legs $a, b$ and hypotenuse $c$, then $a^{2}+b^{2}=c^{2}$
* Area of a square. $A=s^{2}$
* Area of a rectangle. $A=lw$
* Perimeter of a rectangle. $P=2l+2w$
* Volume of a rectangular prism (box). $V=lwh$

Consecutive Integer Problems.

* Consecutive integers are numbers like $-3, -2, 1, 0, …$ or $14, 15, 16$. Notice that they are each different by $1$. Thus, variables for these problems can be $x$, $x+1$, $x+2$, ….
* Examples of consecutive even integers are $8, 10, 12, …$ Notice that these are different by $2$, so appropriate variables are $x$, $x+2$, $x+4$, …
* Odd integers are $1, 3, 5, 7, …$ so just like the even integers, we’ll use $x$, $x+2$, $x+4$, …

Projectile Problems

* The height of a projectile can be modeled as a quadratic function of the time that it is in the air. This comes from the fact that gravity is a constant force. Examples would be tennis balls thrown in the air, volleyballs that are served, cannonballs that are shot, etc.. If we measure the height in feet and the time in seconds, then the equation becomes $h=-16t^{2}+v\_{i}t+h\_{i}$ where $v\_{i}$ is the initial speed in ft/s and $h\_{i}$ is the initial height in ft.

Examples

* The product of two consecutive odd integers is $143$. What are the integers?
* The product of two consecutive negative integers is $56$. What are the integers?
* The sum of the squares of two consecutive even integers is $164$. What are the integers?
* Three consecutive odd integers have the property that the product of the first and third is $7$ greater than $10$ times the second integer. What are the integers?
* The longer leg of a right triangle is $2$ more than twice the shorter leg. Also, the hypotenuse is $1$ more than the longer leg. What are the sides of the triangle?
* Two squares are different in areas by $72$ square units. If their side lengths are different by $4$ units, what are the sides of the squares?
* A person throws a tennis ball in the air while standing on a balcony. The equation that relates the height of the ball to the time in the air is $h=-16t^{2}+20t+20$. What equation could be used to find the time it took the ball to reach $60 ft$? Don’t actually solve the equation. Can you say if there are $0, 1,$ or $2$ solutions to this equation?
* A rectangle has an area that is numerically $3$ times as much as its perimeter (ignore the fact that the units are different). This rectangle has a length that is twice its width. What are the length and width of the rectangle?
* A square box of unknown side length has four equal $2 x 2$ squares removed from its corners, then is folded to make an open box. The volume of the open box is 128 cubic units. What was the original side length of the square and what are the dimensions of the box?