1.4 Notes: Quadratic Equations

**Definition:** Let $a$, $b$, and $c$ be real numbers with $a\ne 0$. Then $ax^{2}+bx+c=0$ is a quadratic equation in $x$ that is in standard form.

There are several methods that can be used to solve quadratic equations. Let’s discuss some of those now.

**Zero-Factor Property:** If $ab=0$, then $a=0$ or $b=0$. In other words, the only way for a product to be zero is if one of its factors is zero.

Practice: Solve each equation using the zero factor property.

* $x^{2}-3x-10=0$
* $4x^{2}-9=0$
* $x^{2}+5x=-6$

**Square Root Property:** If $x^{2}=k$, then $x=\pm \sqrt{k}$. Notice that all quadratic equations that have no linear term can be solved this way.

Practice: Solve each equation using the square root property

* $x^{2}=36$
* $x^{2}+64=0$
* $(3x+1)^{2}=-8$

**Completing the Square:** This process transforms any quadratic equation into one that can be solved by taking square roots. The process is difficult to verbalize, but it centers on recognizing squares of binomials such as $(x+a)^{2}=x^{2}+2ax+a^{2}$ and using the addition property of equality. Suppose we wanted to solve:

$$x^{2}+6x+10=0$$

We should recognize that $(x+3)^{2}=x^{2}+6x+9$ from the above shortcut (You might also do the multiplication from scratch to verify this). This is very close to the equation we want to solve so we’ll separate the $10$ into $9+1$ to make it work. Thus, our equation becomes

* $x^{2}+6x+10=0$
* $\left(x^{2}+6x+9\right)+1=0$
* $(x+3)^{2}+1=0$. This we can solve. It’s a perfect square so we will isolate the square and take square roots to get
* $(x+3)^{2}=-1$
* $x+3=\pm i$
* $x=-3\pm i$

Practice: Solve each equation by completing the square

* $x^{2}-4x+5=0$
* $x^{2}+\frac{2}{3}x-1=0$
* $3x^{2}-5x+6=0$

**The Quadratic Formula:** If one begins with the standard form quadratic equation, $ax^{2}+bx+c=0$ and solves it by completing the square, the solution is $x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$. Thus, this equation can be used to solve any quadratic equation that is written in standard form (this last part is important of course).

Proof of the quadratic equation in class lecture notes:

Practice: Solve each quadratic equation by using the quadratic formula

* $x^{2}+4x+6=0$
* $5x^{2}-3x+2=0$
* $-6x^{2}+8=5x$

Notice that when we use the quadratic formula, the radical term will tell us what set of numbers our solution is a part of. This motivates a new definition. If $a, b, $and $c$ are integers with $ax^{2}+bx+c=0$ and $x=\frac{-b\pm \sqrt{b^{2}-4ac}}{2a}$, then the discriminant, $D=b^{2}-4ac$.

|  |  |  |
| --- | --- | --- |
| Condition | Number of Solutions | Type of Solutions |
| $$D=0$$ | One solution (a double) | Rational |
| $$D<0$$ | 2 | Nonreal Complex |
| $$D>0$$ | 2 | Irrational unless $D$ is the square of an integer and if so, rational. |

Practice: Calculate the discriminant of each quadratic equation and describe the number and type of solutions.

* $-3x^{2}+2=4x$
* $x^{2}-5x=-9$
* $8x+2x^{2}-8=0$

Other Practice: These equations arise from Geometry or Physics and are not really different than those discussed earlier. Just different symbols and more variables.

* Solve $S=4πr^{2}$ for $r$
* Solve $d=\frac{1}{2}gt^{2}+v\_{i}t$ for $t$.