5.7 Properties of Matrices

Today we will focus on how to add matrices, multiply a matrix by a scalar, and multiply a matrix by a matrix.

**Matrix Equality**

* Two matrices are equal if and only if they are the same size and every corresponding entry is equal.

Practice:

Find the values of the variables that make the statement true.

* $\left[\begin{matrix}-3&0\\2&5\end{matrix}\right]=\left[\begin{matrix}3z-5&w\\2x+4&5-3y\end{matrix}\right]$
* $\left[\begin{matrix}2x-6&3w\\2y+9&5-8z\end{matrix}\right]=\begin{matrix}5\\0\\-7\end{matrix}$

**Adding Matrices**

* Addition is defined if and only if both matrices are the same dimensions. This means they must have identical numbers of rows and columns.
* If the above condition is met, simply add corresponding entries to find the new matrix.
* If this condition is not met, then the matrix sum doesn’t exist.
* Subtraction is defined similarly.

Practice:

Let $A=\left[\begin{matrix}-3&0\\2&5\end{matrix}\right]$, $B=\left[\begin{matrix}5&3\\-6&8\end{matrix}\right]$, $C=\begin{matrix}5&-3&-1\\4&0&1\end{matrix}$, $D=\begin{matrix}0&1\\-5&3\\-4&6\end{matrix}$

* $A+B=$
* $A+D=$
* $B+C=$
* $A-B=$

**Multiplying a Matrix by a Scalar**

* A scalar is a real number (could also be complex in future courses).
* Simply multiply every entry of the matrix by the scalar.

Practice:

Using the matrices defined above, calculate:

* $4A$
* $-\frac{2}{3}C$

**Multiplying a Matrix by a Matrix**

* Matrix multiplication is a well-defined process, but is difficult to verbalize so we will do several examples so you can see how the process works.
* Matrix multiplication is defined if and only if the number of columns in the first matrix equals the number of rows in the second matrix.
* If the above condition is met, then the product matrix will have the same number of rows as the first matrix and the same number of columns as the second matrix.
* To find the product matrix calculate the linear combination of each pair of products for each row/column combination. This can be written out mathematically, but will be much easier to understand if we do some examples.

Using the matrices defined above, calculate:

* $AB=$
* $BA=$
* $AC=$
* $CA=$
* $CD=$
* $DC=$